AzMERIT | Arizona's Statewide Achievement Assessment for English Language Arts and Mathematics

Math Item Specifications

HIGH SCHOOL (STATISTICS AND PROBABILITY)

Arizona Department of Education with American Institutes for Research - 2016

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Introduction

The Arizona Statewide Achievement Assessment for English Language Arts and Mathematics (AzMERIT) is Arizona's statewide achievement test. AzMERIT assesses the Arizona College and Career Ready Standards (AzCCRS) adopted by the Arizona State Board of Education in 2010. AzMERIT will inform students, teachers, and parents about preparedness for college and careers upon graduating from high school. AzMERIT tests are computer-based, meaning that they can better assess students' critical thinking skills and provide them with opportunities to demonstrate a deeper understanding of the materials. Computer-based testing also allows for the use of a variety of innovative items types.

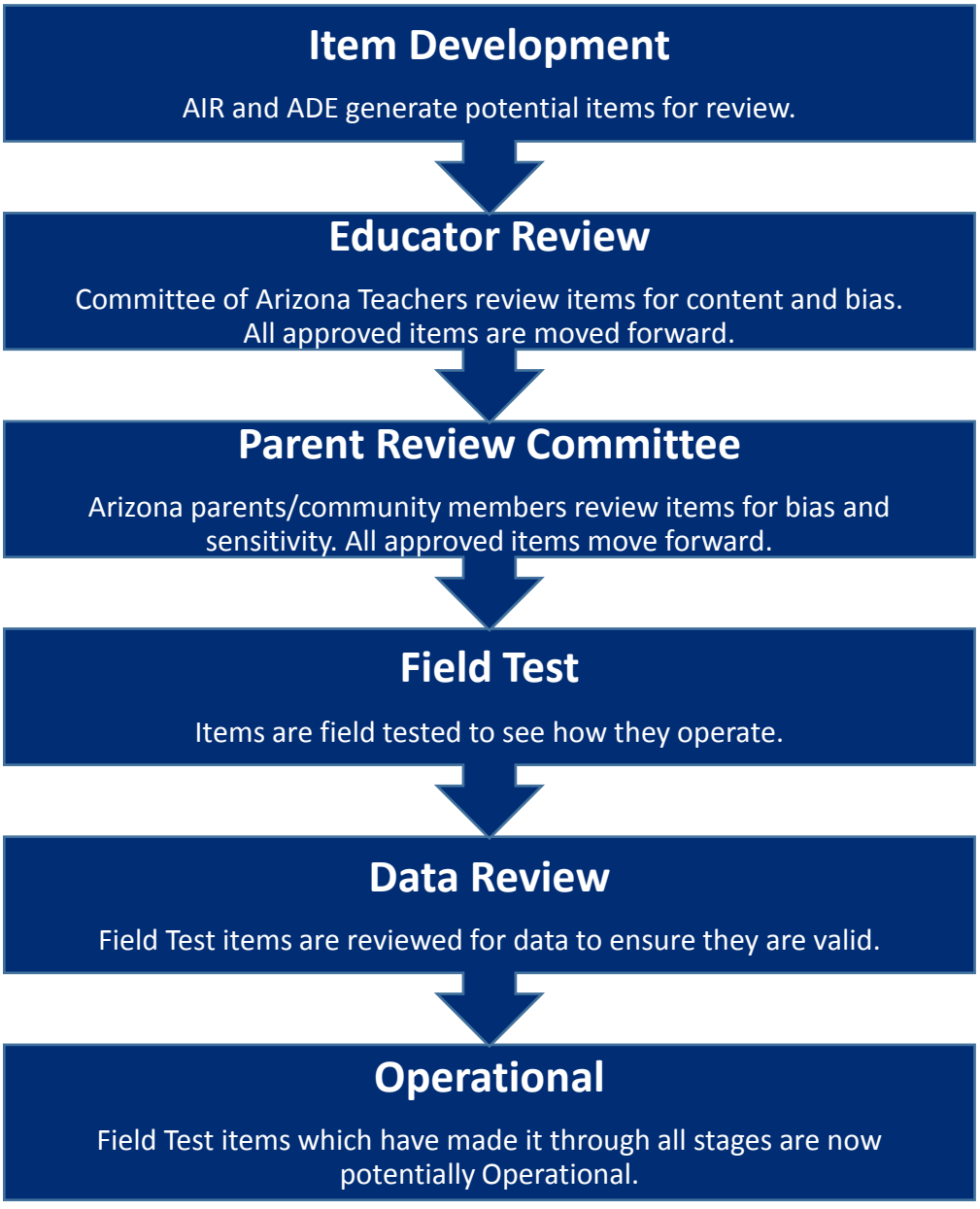
During the item-development process, all AzMERIT items are written in accordance with the Item Specifications and are reviewed and approved by a committee of Arizona educators to confirm alignment and appropriateness for inclusion in the test. AzMERIT items are generally representative of Arizona's geographic regions and culturally diverse population. Items are reviewed for the following kinds of bias: gender, racial, ethnic, linguistic, religious, geographic, and socioeconomic. Item reviews also include consideration of issues related to individuals with disabilities. Arizona community members also have an opportunity to review items for issues of potential concern to members of the community at large. Reviewers are asked to consider the variety of cultural, regional, philosophical, political, and religious backgrounds throughout Arizona, and then to determine whether the subject matter will be acceptable to Arizona students, families, and other members of Arizona communities.

This *AzMERIT Item Specifications* is a resource document that defines the content and format of the test and test items for item writers and reviewers. Each *Item Specifications* document indicates the alignment of items with the AzCCRS. It also serves to provide all stakeholders with information about the scope and function of assessment items. This document can also serve to assist educators to understand how assessment items are developed in alignment with the standards for English language arts and math. These item specifications for AzMERIT are intended to provide information regarding standards, item formats and response types. The descriptions of math practices, blueprints, and depth of knowledge in this document are meant to provide an overview of the test. Item specifications are meant for the purposes of assessment, not instruction. They are not intended to be tools for instruction or the basis for curricula. AzMERIT has a test blueprint that was developed by Arizona and is different from any other state or consortium test blueprint.

For the math portion of AzMERIT, all of the test questions are aligned to the mathematic content standards for these subject areas. Similarly, each item assesses a single domain and aligns to one or more of the eight Math Practices. Any item specifications that are absent for standards listed in this document may be under development. This document does not endorse the exclusion of the instruction of any grade-level content standards. The test will ask questions that check a student's conceptual understanding of math as well as their procedural skills. These items have been written to be free from bias and sensitivity, and widely vary in their degree of difficulty.

Item Development Process

AzMERIT items go through a rigorous review before they are operational. When an item is "operational" it means it is used to determine a student's score on the assessment. This is a description of the process every item must go through before it is operational on AzMERIT.



Sample tests are available online for the math portion of AzMERIT. For more information view the Guide to the Sample Tests at www.azmeritportal.org.

Test Construction Guidelines

The construction of the AzMERIT assessment is guided by the depth and rigor of the Arizona College and Career Ready Standards. Items are created to address key components of the standards and assess a range of important skills. The AzMERIT Blueprint provides an overview of the distribution of items on the AzMERIT according to the standards. The standards for Math Practices are embedded within all AzMERIT items. Further, the AzMERIT blueprint outlines the Depth of Knowledge distribution of items.

Math Practices

The standards for Mathematical Practice highlight the knowledge, skills and abilities that should be developed in students at all grades. The Mathematical Practices are a part of each course description for Grades 3 through 8, Algebra I, Geometry, and Algebra 2. These practices are a vital part of the curriculum. These skills are often difficult to measure, and as a result, every item created for AzMERIT aligns to one or more of the following eight Mathematical Practices.

Math Practice (MP)	Description
Math Practice 1	**Make sense of problems and persevere in solving them.** Mathematically proficient students start by explaining to themselves the meaning of a problem and looking for entry points to its solution. They analyze givens, constraints, relationships, and goals. They make conjectures about the form and meaning of the solution and plan a solution pathway rather than simply jumping into a solution attempt. They consider analogous problems, and try special cases and simpler forms of the original problem in order to gain insight into its solution. They monitor and evaluate their progress and change course if necessary. Older students might, depending on the context of the problem, transform algebraic expressions or change the viewing window on their graphing calculator to get the information they need. Mathematically proficient students can explain correspondences between equations, verbal descriptions, tables, and graphs or draw diagrams of important features and relationships, graph data, and search for regularity or trends. Younger students might rely on using concrete objects or pictures to help conceptualize and solve a problem. Mathematically proficient students check their answers to problems using a different method, and they continually ask themselves, “Does this make sense?” They can understand the approaches of others to solving complex problems and identify correspondences between different approaches.

Math Practice (MP)	Description
Math Practice 2	**Reason abstractly and quantitatively**. Mathematically proficient students make sense of quantities and their relationships in problem situations. They bring two complementary abilities to bear on problems involving quantitative relationships: the ability to decontextualize—to abstract a given situation and represent it symbolically and manipulate the representing symbols as if they have a life of their own, without necessarily attending to their referents—and the ability to contextualize, to pause as needed during the manipulation process in order to probe into the referents for the symbols involved. Quantitative reasoning entails habits of creating a coherent representation of the problem at hand; considering the units involved; attending to the meaning of quantities, not just how to compute them; and knowing and flexibly using different properties of operations and objects.
Math Practice 3	**Construct viable arguments and critique the reasoning of others.** Mathematically proficient students understand and use stated assumptions, definitions, and previously established results in constructing arguments. They make conjectures and build a logical progression of statements to explore the truth of their conjectures. They are able to analyze situations by breaking them into cases, and can recognize and use counterexamples. They justify their conclusions, communicate them to others, and respond to the arguments of others. They reason inductively about data, making plausible arguments that take into account the context from which the data arose. Mathematically proficient students are also able to compare the effectiveness of two plausible arguments, distinguish correct logic or reasoning from that which is flawed, and—if there is a flaw in an argument—explain what it is. Elementary students can construct arguments using concrete referents such as objects, drawings, diagrams, and actions. Such arguments can make sense and be correct, even though they are not generalized or made formal until later grades. Later, students learn to determine domains to which an argument applies. Students at all grades can listen or read the arguments of others, decide whether they make sense, and ask useful questions to clarify or improve the arguments.

Math Practice (MP)	Description
Math Practice 4	**Model with mathematics.** Mathematically proficient students can apply the mathematics they know to solve problems arising in everyday life, society, and the workplace. In early grades, this might be as simple as writing an addition equation to describe a situation. In middle grades, a student might apply proportional reasoning to plan a school event or analyze a problem in the community. By high school, a student might use geometry to solve a design problem or use a function to describe how one quantity of interest depends on another. Mathematically proficient students who can apply what they know are comfortable making assumptions and approximations to simplify a complicated situation, realizing that these may need revision later. They are able to identify important quantities in a practical situation and map their relationships using such tools as diagrams, two-way tables, graphs, flowcharts and formulas. They can analyze those relationships mathematically to draw conclusions. They routinely interpret their mathematical results in the context of the situation and reflect on whether the results make sense, possibly improving the model if it has not served its purpose.
Math Practice 5	**Use appropriate tools strategically.** Mathematically proficient students consider the available tools when solving a mathematical problem. These tools might include pencil and paper, concrete models, a ruler, a protractor, a calculator, a spreadsheet, a computer algebra system, a statistical package, or dynamic geometry software. Proficient students are sufficiently familiar with tools appropriate for their grade or course to make sound decisions about when each of these tools might be helpful, recognizing both the insight to be gained and their limitations. For example, mathematically proficient high school students analyze graphs of functions and solutions generated using a graphing calculator. They detect possible errors by strategically using estimation and other mathematical knowledge. When making mathematical models, they know that technology can enable them to visualize the results of varying assumptions, explore consequences, and compare predictions with data. Mathematically proficient students at various grade levels are able to identify relevant external mathematical resources, such as digital content located on a website, and use them to pose or solve problems. They are able to use technological tools to explore and deepen their understanding of concepts.

Math Practice (MP)	Description
Math Practice 6	**Attend to precision.** Mathematically proficient students try to communicate precisely to others. They try to use clear definitions in discussion with others and in their own reasoning. They state the meaning of the symbols they choose, including using the equal sign consistently and appropriately. They are careful about specifying units of measure, and labeling axes to clarify the correspondence with quantities in a problem. They calculate accurately and efficiently, express numerical answers with a degree of precision appropriate for the problem context. In the elementary grades, students give carefully formulated explanations to each other. By the time they reach high school they have learned to examine claims and make explicit use of definitions.
Math Practice 7	**Look for and make use of structure.** Mathematically proficient students look closely to discern a pattern or structure. Young students, for example, might notice that three and seven more is the same amount as seven and three more, or they may sort a collection of shapes according to how many sides the shapes have. Later, students will see 7×8 equals the well-remembered $7 \times 5 + 7 \times 3$, in preparation for learning about the distributive property. In the expression $x^2 + 9x + 14$, older students can see the 14 as 2×7 and the 9 as $2 + 7$. They recognize the significance of an existing line in a geometric figure and can use the strategy of drawing an auxiliary line for solving problems. They also can step back for an overview and shift perspective. They can see complicated things, such as some algebraic expressions, as single objects or as being composed of several objects. For example, they can see $5 – 3(x – y)^2$ as 5 minus a positive number times a square and use that to realize that its value cannot be more than 5 for any real numbers x and y.
Math Practice 8	**Look for and express regularity in repeated reasoning.** Mathematically proficient students notice if calculations are repeated, and look both for general methods and for shortcuts. Upper elementary students might notice when dividing 25 by 11 that they are repeating the same calculations over and over again, and conclude they have a repeating decimal. By paying attention to the calculation of slope as they repeatedly check whether points are on the line through (1, 2) with slope 3, middle school students might abstract the equation $(y – 2)/(x – 1) = 3$. Noticing the regularity in the way terms cancel when expanding $(x – 1)(x + 1)$, $(x – 1)(x^2 + x + 1)$, and $(x – 1)(x^3 + x^2 + x + 1)$ might lead them to the general formula for the sum of a geometric series. As they work to solve a problem, mathematically proficient students maintain oversight of the process, while attending to the details. They continually evaluate the reasonableness of their intermediate results.

Blueprint

The AzMERIT blueprints detail specific information in regard to the domains tested at each grade level. The blueprint outlines the percentage of points aligned to each domain.

	Domain	Minimum	Maximum
Algebra 1	Algebra	40%	44%
	Functions	36%	40%
	Statistics	17%	21%

Approximately 70% of the assessment for High School will be on major content.

	Domain	Minimum	Maximum
Geometry	Congruence	23%	27%
	Similarity, Right Triangles, And Trigonometry	27%	31%
	Circles, Geometric Measurement and Dimensions	23%	27%
	Modeling with Geometry	17%	21%

Approximately 70% of the assessment for High School will be on major content.

	Domain	Minimum	Maximum
Algebra 2	Algebra	34%	38%
	Functions	32%	36%
	Statistics	27%	31%

Approximately 70% of the assessment for High School will be on major content.

Depth of Knowledge (DOK)

DOK refers to the level of rigor or sophistication of the task in a given item, designed to reflect the complexity of the AzCCRS. Items at DOK level 1 focus on the recall of information, such as definitions, terms, and simple procedures. Items at DOK 2 require students to make decisions, solve problems, or recognize patterns; in general, they require a greater degree of engagement and cognitive processing than items at DOK 1. Items at DOK 3 feature higher-order cognitive tasks that assess students' capacities to approach abstract or complex problems.

Percentage of Points by Depth of Knowledge (DOK) Level			
High School	DOK Level 1	DOK Level 2	DOK Level 3
	10% - 20%	60% - 70%	12% - 30%

For more information on DOK go to www.azed.gov/AzMERIT.

Calculators

Calculators are permitted for both the paper-based and computer-based assessment for High School Math.

Item Formats

The AzMERIT Assessments are composed of item formats that include traditional multiple-choice response items and technology-enhanced response items (TEI). TEIs are computer-delivered response items that require students to interact with test content to select, construct, and/or support their responses. TEIs are better able to assess a deeper level of understanding.

Currently, there are nine types of TEIs that may appear on the High School Math computer based assessment for AzMERIT:

- Editing Tasks (ET)
- Editing Task Choice (ETC)
- Equation Editor (EQ)
- Graphic Response Item Display (GRID)
- Hot Text (HT)
    - Selectable Hot Text
    - Drag-and-Drop Hot Text
- Matching Item (MI)
- Multi-Select (MS)
- Open Response
- Table Item (TI)

For paper based assessments (including those for students with an IEP or 504 plan that specifies a paper based accommodation), TEIs will be modified so that they can be scanned and scored electronically or hand-scored.

See the table below for a description of each TEI. In addition, for examples of each response item format described, see the AzMERIT Training Tests at www.azmeritportal.org.

Item Format	Description
Editing Task (ET)	The student clicks on a highlighted word or phrase that may be incorrect, which reveals a text box. The directions in the text box direct the student to replace the highlighted word or phrase with the correct word or phrase. For paper-based assessments, this item type may be replaced with another item type that assesses the same standard and can be scanned and scored electronically.
Editing Task Choice (ETC)	The student clicks a highlighted word or phrase, which reveals a drop-down menu containing options for correcting an error as well as the highlighted word or phrase as it is shown in the sentence to indicate that no correction is needed. The student then selects the correct word or phrase from the drop-down menu. For paper-based assessments, the item is modified so that it can be scanned and scored electronically. The student fills in a circle to indicate the correct word or phrase.

Item Format	Description
Equation Editor (EQ)	The student is presented with a toolbar that includes a variety of mathematical symbols that can be used to create a response. Responses may be in the form of a number, variable, expression, or equation, as appropriate to the test item. For paper-based assessments, this item type may be replaced with a modified version of the item that can be scanned and scored electronically or replaced with another item type that assesses the same standard and can be scanned and scored electronically.
Graphic Response Item Display (GRID)	The student selects numbers, words, phrases, or images and uses the drag-and-drop feature to place them into a graphic. This item type may also require the student to use the point, line, or arrow tools to create a response on a graph. For paper-based assessments, this item type may be replaced with another item type that assesses the same standard and can be scanned and scored electronically.
Hot Text (HT)	**Selectable Hot Text -** Excerpted sentences from the text are presented in this item type. When the student hovers over certain words, phrases, or sentences, the options highlight. This indicates that the text is selectable ("hot"). The student can then click on an option to select it. For paper- based assessments, a "selectable" hot text item is modified so that it can be scanned and scored electronically. In this version, the student fills in a circle to indicate a selection.
	Drag-and-Drop Hot Text - Certain numbers, words, phrases, or sentences may be designated "draggable" in this item type. When the student hovers over these areas, the text highlights. The student can then click on the option, hold down the mouse button, and drag it to a graphic or other format. For paper-based assessments, drag-and-drop hot text items will be replaced with another item type that assesses the same standard and can be scanned and scored electronically.
Matching Item (MI)	The student checks a box to indicate if information from a column header matches information from a row. For paper-based assessments, this item type may be replaced with another item type that assesses the same standard and can be scanned and scored electronically.
Multi-Select (MS)	The student is directed to select all of the correct answers from among a number of options. These items are different from multiple-choice items, which allow the student to select only one correct answer. These items appear in the online and paper-based assessments.
Open Response	The student uses the keyboard to enter a response into a text field. These items can usually be answered in a sentence or two. For paper-based assessments, this item type may be replaced with another item type that assesses the same standard and can be scanned and scored electronically.

Item Format	Description
Table Item (TI)	The student types numeric values into a given table. The student may complete the entire table or portions of the table depending on what is being asked. For paper-based assessments, this item type may be replaced with another item type that assesses the same standard and can be scanned and scored electronically.

Arizona's College and Career Ready Standards (AzCCRS)

Statistics and Probability

Conditional Probability & the Rules of Probability (S-CP)

HS.S-CP.A – Understand independence and conditional probability and use them to interpret data.

HS.S-CP.B – Use the rules of probability to compute probabilities of compound events.

Making Inferences & Justifying Conclusions (S-IC)

HS.S-IC.A – Understand and evaluate random processes underlying statistical experiments.

HSS-IC.B – Make inferences and justify conclusions from sample surveys, experiments, and observational studies.

Interpreting Categorical & Quantitative Data (S-ID)

HS.S-ID.A – Summarize, represent, and interpret data on a single count of measurement variable.

HS.S-ID.B – Summarize, represent and interpret data on two categorical and quantitative variables.

HS.S-ID.C – Interpret linear models.

Using Probability to Make Decisions (S-MD)

HS.S-MD.A – Calculate expected values and use them to solve problems.

HS.S-MD.B – Use probability to evaluate outcomes of decisions.

High School Math Item Specifications (Statistics and Probability)

Conditional Probability & the Rules of Probability

Content Standards	AzCCRS.Math.Content.S-CP.A.1 Describe events as subsets of a sample space (the set of outcomes) using characteristics (or categories) of the outcomes, or as unions, intersections, or complements of other events ("or," "and," "not").		
Explanations	Intersection: The intersection of two sets A and B is the set of elements that are common to both set A and set B. It is denoted by A ∩ B and is read 'A intersection B.' Union: The union of two sets A and B is the set of elements, which are in A or in B or in both. It is denoted by A ∪ B and is read 'A union B.' Complement: The complement of the set A ∪ B is the set of elements that are members of the universal set U but are not in A ∪ B. It is denoted by (A ∪ B)'		
Content Limits	This standard is aligned to Algebra II only. Positive rational numbers		
Common Item Formats	The Item Formats section on pages 11 through 13 provides a list of item formats that may be used to assess this standard. The common item formats include but are not limited to those shown with the sample task demands.		
Context	Context is allowed.	**Math Practices**	2, 4, 6, 7

Sample Task Demands	**Common Item Formats**	**Recommended Math Practices**
Students will be required to identify events as outcomes of a trial.	• Multiple Choice Response • Multi-Select Response	2, 6, 7
Students will be required to identify multiple events as subsets of the sample space, including unions, intersections, and complements.		2, 4, 6, 7

Content Standards	AzCCRS.Math.Content.S-CP.A.2 Understand that two events A and B are independent if the probability of A and B occurring together is the product of their probabilities, and use this characterization to determine if they are independent.		
Explanations	None		
Content Limits	This standard is aligned to Algebra II only. Positive rational numbers		
Common Item Formats	The Item Formats section on pages 11 through 13 provides a list of item formats that may be used to assess this standard. The common item formats include but are not limited to those shown with the sample task demands.		
Context	Context is allowed.	**Math Practices**	2, 4, 6, 7

Sample Task Demands	**Common Item Formats**	**Recommended Math Practices**
Students will be required to identify independent events given their probabilities.	• Equation Response • Multiple Choice Response • Multi-Select Response	2, 6, 7
Students will be required to determine the probability of the other event given that two events are independent and the probability of one event.		2, 6, 7
Students will be required to interpret two events in terms of independence given the probabilities of the two events.		2, 4, 6, 7

Content Standards	AzCCRS.Math.Content.S-CP.A.3 Understand the conditional probability of A given B as P(A and B)/P(B), and interpret independence of A and B as saying that the conditional probability of A given B is the same as the probability of A, and the conditional probability of B given A is the same as the probability of B.		
Explanations	None		
Content Limits	This standard is aligned to Algebra II only. Rational positive numbers		
Common Item Formats	The Item Formats section on pages 11 through 13 provides a list of item formats that may be used to assess this standard. The common item formats include but are not limited to those shown with the sample task demands.		
Context	Context is allowed.	**Math Practices**	2, 4, 6, 7

Sample Task Demands	Common Item Formats	Recommended Math Practices
Students will be required to compute conditional probabilites.	• Equation Response • Multiple Choice Response • Multi-Select Response	2, 6, 7
Students will be required to identify independent events given their probabilities and one conditional probability.		2, 6, 7
Students will be required to determine the conditional probability (or vice versa) given that two events are independent and the probability of one event.		2, 6, 7
Students will be required to interpret the events in terms of independence given the probability of an event and the conditional probability of that event with another event.		2, 4, 6, 7

<table>
<tr><td>Content Standards</td><td colspan="3">AzCCRS.Math.Content.S-CP.A.4 Construct and interpret two-way frequency tables of data when two categories are associated with each object being classified. Use the two-way table as a sample space to decide if events are independent and to approximate conditional probabilities.</td></tr>
<tr><td>Explanations</td><td colspan="3">None</td></tr>
<tr><td>Content Limits</td><td colspan="3">This standard is aligned to Algebra II only.</td></tr>
<tr><td>Common Item Formats</td><td colspan="3">The Item Formats section on pages 11 through 13 provides a list of item formats that may be used to assess this standard. The common item formats include but are not limited to those shown with the sample task demands.</td></tr>
<tr><td>Context</td><td>Context is allowed.</td><td>Math Practices</td><td>1, 2, 3, 4, 5, 6, 7, 8</td></tr>
<tr><td colspan="2">Sample Task Demands</td><td>Common Item Formats</td><td>Recommended Math Practices</td></tr>
<tr><td colspan="2">Students will be required to determine independence of events or conditional probabilities given a two-way table.</td><td rowspan="2">• Equation Response
• Multiple Choice Response
• Table Response</td><td>1, 2, 4, 5, 7, 8</td></tr>
<tr><td colspan="2">Students will be required to complete a two-way table to satisfy criteria related to independence or conditional probability.</td><td>1, 2, 3, 4, 5, 6, 7, 8</td></tr>
</table>

<table>
<tr><td>Content Standards</td><td colspan="3">AzCCRS.Math.Content.S-CP.A.5 Recognize and explain the concepts of conditional probability and independence in everyday language and everyday situations.</td></tr>
<tr><td>Explanations</td><td colspan="3">None</td></tr>
<tr><td>Content Limits</td><td colspan="3">This standard is aligned to Algebra II only.
Rational positive numbers</td></tr>
<tr><td>Common Item Formats</td><td colspan="3">The Item Formats section on pages 11 through 13 provides a list of item formats that may be used to assess this standard. The common item formats include but are not limited to those shown with the sample task demands.</td></tr>
<tr><td>Context</td><td>Context is allowed.</td><td>Math Practices</td><td>1, 4, 6, 8</td></tr>
<tr><td colspan="2">Sample Task Demands</td><td>Common Item Formats</td><td>Recommended Math Practices</td></tr>
<tr><td colspan="2">Students will be required to identify examples of independence and conditional probability given a scenario.</td><td rowspan="2">• Multiple Choice Response</td><td>1, 8</td></tr>
<tr><td colspan="2">Students will be required to interpret this in terms of the context given that two events are independent or a conditional probability.</td><td>1, 4, 6, 8</td></tr>
</table>

Content Standards	AzCCRS.Math.Content.S-CP.B.6 Find the conditional probability of A given B as the fraction of B's outcomes that also belong to A, and interpret the answer in terms of the model.		
Explanations	None		
Content Limits	This standard is aligned to Algebra II only. Data is given by raw data and not probabilities		
Common Item Formats	The Item Formats section on pages 11 through 13 provides a list of item formats that may be used to assess this standard. The common item formats include but are not limited to those shown with the sample task demands.		
Context	Context is allowed.	**Math Practices**	1, 4, 5, 7

Sample Task Demands	**Common Item Formats**	**Recommended Math Practices**
Students will be required to compute a conditional probability.	• Equation Response • Multiple Choice Response	1, 5, 7
Students will be required to find pieces of raw data given a conditional probability and some other raw data.		1, 4, 5, 7
Students will be required to interpret the meaning of the conditional probability given data and a conditional probability.		1, 4, 5, 7

<table>
<tr><td>Content Standards</td><td colspan="3">AzCCRS.Math.Content.S-CP.B.7 Apply the Addition Rule, P(A or B) = P(A) + P(B) — P(A and B), and interpret the answer in terms of the model.</td></tr>
<tr><td>Explanations</td><td colspan="3">None</td></tr>
<tr><td>Content Limits</td><td colspan="3">This standard is aligned to Algebra II only.</td></tr>
<tr><td>Common Item Formats</td><td colspan="3">The Item Formats section on pages 11 through 13 provides a list of item formats that may be used to assess this standard. The common item formats include but are not limited to those shown with the sample task demands.</td></tr>
<tr><td>Context</td><td>Context is allowed.</td><td>Math Practices</td><td>4, 5, 6, 7</td></tr>
<tr><td colspan="2">Sample Task Demands</td><td>Common Item Formats</td><td>Recommended Math Practices</td></tr>
<tr><td colspan="2">Students will be required to compute the probability of the union of two events.</td><td rowspan="3">• Equation Response
• Proposition Response</td><td>4, 5, 6, 7</td></tr>
<tr><td colspan="2">Students will be required to find a missing probability using the Addition Rule given the probability of the union of two events and other probabilities.</td><td>4, 5, 6, 7</td></tr>
<tr><td colspan="2">Students will be required to interpret the meaning of the union given a context and the probability of the union of two events.</td><td>4, 5, 6, 7</td></tr>
</table>

Making Inferences & Justifying Conclusions

Content Standards	AzCCRS.Math.Content.S-IC.A.1 Understand statistics as a process for making inferences about population parameters based on a random sample from that population.		
Explanations	None		
Content Limits	This standard is aligned to Algebra II only. Quantities should be simple and realistic to the context so as to allow the student to show knowledge of the concept rather than computational skills.		
Common Item Formats	The Item Formats section on pages 11 through 13 provides a list of item formats that may be used to assess this standard. The common item formats include but are not limited to those shown with the sample task demands.		
Context	Context is allowed.	**Math Practices**	4, 6

Sample Task Demands	**Common Item Formats**	**Recommended Math Practices**
Students will be required to order steps in a statistical process or select a sample that represents a given population.	• Hot Text Response • Multiple Choice Response	4, 6
Students will be required to describe flaws in a statistical process (i.e., not random) or recommend a correct course of action.		4, 6

<table>
<tr><td>Content Standards</td><td colspan="3">AzCCRS.Math.Content.S-IC.A.2 Decide if a specified model is consistent with results from a given data-generating process, e.g., using simulation.</td></tr>
<tr><td>Explanations</td><td colspan="3">Possible data-generating processes include (but are not limited to): flipping coins, spinning spinners, rolling a number cube, and simulations using the random number generators.

The law of large numbers states that as the sample size increases, the experimental probability will approach the theoretical probability. Comparison of data from repetitions of the same experiment is part of the model building verification process.</td></tr>
<tr><td>Content Limits</td><td colspan="3">This standard is aligned to Algebra II only.

Quantities should be simple and realistic to the context so as to allow the student to show knowledge of the concept rather than computational skills.</td></tr>
<tr><td>Common Item Formats</td><td colspan="3">The Item Formats section on pages 11 through 13 provides a list of item formats that may be used to assess this standard. The common item formats include but are not limited to those shown with the sample task demands.</td></tr>
<tr><td>Context</td><td>Context is allowed.</td><td>Math Practices</td><td>1, 2, 3, 4, 5, 6, 7, 8</td></tr>
<tr><td colspan="2">Sample Task Demands</td><td>Common Item Formats</td><td>Recommended Math Practices</td></tr>
<tr><td colspan="2">Students will be required to predict the most likely results of a simulation given a probability model.</td><td rowspan="2">• Equation Response
• Multiple Choice Response</td><td>1, 2, 4, 5, 7, 8</td></tr>
<tr><td colspan="2">Students will be required to select the best probability model given the results of a simulation.</td><td>1, 2, 3, 4, 5, 6, 7, 8</td></tr>
</table>

Content Standards	AzCCRS.Math.Content.S-IC.B.3 Recognize the purposes of and differences among sample surveys, experiments, and observational studies; explain how randomization relates to each.		
Explanations	Students should be able to explain techniques/applications for randomly selecting study subjects from a population and how those techniques/applications differ from those used to randomly assign existing subjects to control groups or experimental groups in a statistical experiment. In statistics, an observational study draws inferences about the possible effect of a treatment on subjects, where the assignment of subjects into a treated group versus a control group is outside the control of the investigator (for example, observing data on academic achievement and socio-economic status to see if there is a relationship between them). This is in contrast to controlled experiments, such as randomized controlled trials, where each subject is randomly assigned to a treated group or a control group before the start of the treatment.		
Content Limits	This standard is aligned to Algebra II only.		
Common Item Formats	The Item Formats section on pages 11 through 13 provides a list of item formats that may be used to assess this standard. The common item formats include but are not limited to those shown with the sample task demands.		
Context	Context is allowed.	**Math Practices**	3, 4, 6

Sample Task Demands	**Common Item Formats**	**Recommended Math Practices**
Students will be required to identify a given activity as a survey sample, experiment, or observational study	• Multiple Choice Response	3, 6
Students will be required to identify a substantive difference between the three types of activities, or the most appropriate activity for a given research question.		3, 6
Students will be required to design a study that correctly applies randomization given specific criteria.		3, 4, 6

<table>
<tr><td>Content Standards</td><td colspan="3">AzCCRS.Math.Content.S-IC.B.4 Use data from a sample survey to estimate a population mean or proportion; develop a margin of error through the use of simulation models for random sampling.</td></tr>
<tr><td>Explanations</td><td colspan="3">None</td></tr>
<tr><td>Content Limits</td><td colspan="3">This standard is aligned to Algebra II only.

The student should not be required to compute a margin of error, but identify what factors lead to larger or smaller margins of error</td></tr>
<tr><td>Common Item Formats</td><td colspan="3">The Item Formats section on pages 11 through 13 provides a list of item formats that may be used to assess this standard. The common item formats include but are not limited to those shown with the sample task demands.</td></tr>
<tr><td>Context</td><td>Context is allowed.</td><td>Math Practices</td><td>1, 4, 5</td></tr>
<tr><td colspan="2">Sample Task Demands</td><td>Common Item Formats</td><td>Recommended Math Practices</td></tr>
<tr><td colspan="2">Students will be required to provide a sample mean given raw data.</td><td rowspan="2">• Equation Response
• Proposition Response
• Simulator Response</td><td>1, 4, 5</td></tr>
<tr><td colspan="2">Students will be required to relate margin of error to the characteristics of the sample population and the survey methodology.</td><td>1, 4, 5</td></tr>
</table>

Content Standards	AzCCRS.Math.Content.S-IC.B.5 Use data from a randomized experiment to compare two treatments; use simulations to decide if differences between parameters are significant.		
Explanations	Treatment is a term used in the context of an experimental design to refer to any prescribed combination of values of explanatory variables. For example, one wants to determine the effectiveness of weed killer. Two equal parcels of land in a neighborhood are treated; one with a placebo and one with weed killer to determine whether there is a significant difference in effectiveness in eliminating weeds.		
Content Limits	This standard is aligned to Algebra II only. Items should give students two populations (people given a medication, people not given a medication) and data on those two populations (sample mean of recovery).		
Common Item Formats	The Item Formats section on pages 11 through 13 provides a list of item formats that may be used to assess this standard. The common item formats include but are not limited to those shown with the sample task demands.		
Context	Context is allowed.	**Math Practices**	1, 4, 5, 8

Sample Task Demands	**Common Item Formats**	**Recommended Math Practices**
Students will be required to give a quantitative likelihood that two treatments are significantly different	• Equation Response • Proposition Response • Simulator Response	1, 4, 5, 8
Students will be required to describe to what degree the results of a particular randomized experiment answer a given research question.		1, 4, 5, 8
Students will be required to reach conclusions about the effectiveness of a treatment based on given data.		1, 4, 5, 8

Content Standards	AzCCRS.Math.Content.S-IC.B.6 Evaluate reports based on data.		
Explanations	Explanations can include but are not limited to sample size, biased survey sample, interval scale, unlabeled scale, uneven scale, and outliers that distort the line-of-best-fit. In a pictogram the symbol scale used can also be a source of distortion. As a strategy, collect reports published in the media and ask students to consider the source of the data, the design of the study, and the way the data are analyzed and displayed.		
Content Limits	This standard is aligned to Algebra II only. If approximations using the normal curve are to be used by the student, a standard normal table or equivalent should be provided. Student must be given data and a conclusion to evaluate.		
Common Item Formats	The Item Formats section on pages 11 through 13 provides a list of item formats that may be used to assess this standard. The common item formats include but are not limited to those shown with the sample task demands.		
Context	Context is allowed.	**Math Practices**	1, 2, 3, 4, 5, 6, 7, 8
Sample Task Demands		**Common Item Formats**	**Recommended Math Practices**
Students will be required to evaluate the validity of a claim derived from the results of a randomized experiment.		• Equation Response • Multiple Choice Response	1, 2, 3, 4, 5, 6, 7, 8

Interpreting Categorical & Quantitative Data

Content Standards	AzCCRS.Math.Content.S-ID.A.1 Represent data with plots on the real number line (dot plots, histograms, and box plots).		
Explanations	None		
Content Limits	This standard is aligned to Algebra I only. The amount of data to be plotted should be reasonable.		
Common Item Formats	The Item Formats section on pages 11 through 13 provides a list of item formats that may be used to assess this standard. The common item formats include but are not limited to those shown with the sample task demands.		
Context	Context is allowed.	**Math Practices**	4, 5
Sample Task Demands		**Common Item Formats**	**Recommended Math Practices**
Students will be required to construct a data display.		• Graphic Response • Multiple Choice Response	4, 5

Content Standards	AzCCRS.Math.Content.S-ID.A.2 Use statistics appropriate to the shape of the data distribution to compare center (median, mean) and spread (interquartile range, standard deviation) of two or more different data sets. AzCCRS.Math.Content.S-ID.A.3 Interpret differences in shape, center, and spread in the context of the data sets, accounting for possible effects of extreme data points (outliers).		
Explanations	None		
Content Limits	This standard is aligned to Algebra I only.		
Common Item Formats	The Item Formats section on pages 11 through 13 provides a list of item formats that may be used to assess this standard. The common item formats include but are not limited to those shown with the sample task demands.		
Context	Context is allowed.	**Math Practices**	2, 3, 4, 5, 7

Sample Task Demands	**Common Item Formats**	**Recommended Math Practices**
Students will be required to identify data distributions that share commonalities (i.e., same spread, interquartile range, median, and mean) through inspection.	• Equation Response • Graphic Response • Multiple Choice Response • Multi-Select Response	2, 4, 5, 7
Students will be required to distinguish between different spreads to compare the mean and medians of the data set.		2, 4, 5, 7
Students will be required to construct a graph given information about the shape, center, and spread.		2, 4, 5, 7
Students will be required to compare different distributions in order to draw conclusions about the effects of an extreme outlier on different spreads		2, 3, 4, 5, 7
Students will be required to make inferences about the spread of distributions to draw conclusions about the given context. (i.e., what does a skewed distribution of test scores tell us about the test questions).		2, 3, 4, 5, 7

Content Standards	AzCCRS.Math.Content.S-ID.A.4 Use the mean and standard deviation of a data set to fit it to a normal distribution and to estimate population percentages. Recognize that there are data sets for which such a procedure is not appropriate. Use calculators, spreadsheets, and tables to estimate areas under the normal curve.		
Explanations	None		
Content Limits	This standard is aligned to Algebra II only. If a student is required to estimate a population percentage not associated with 1, 2, or 3 standard deviations from the mean, a normal table or some equivalent mechanism must be provided. Items should state that the data are approximately normally distri		
Common Item Formats	The Item Formats section on pages 11 through 13 provides a list of item formats that may be used to assess this standard. The common item formats include but are not limited to those shown with the sample task demands.		
Context	Context is allowed.	**Math Practices**	1, 2, 3, 4, 5, 6, 7, 8

Sample Task Demands	Common Item Formats	Recommended Math Practices
Students will be required to identify a population percentage or area under a curve within 1, 2, or 3 standard deviations.	• Equation Response • Graphic Response • Multiple Choice Response	1, 2, 4, 5, 6, 7, 8
Students will be required to identify a population percentage for standard deviations other than 1, 2, or 3.		1, 2, 4, 5, 6, 7, 8
Students will be required to explain why a data set should or should not be modeled with a normal distribution.		1, 2, 3, 4, 5, 6, 7, 8

Content Standards	AzCCRS.Math.Content.S-ID.B.5 Summarize categorical data for two categories in two-way frequency tables. Interpret relative frequencies in the context of the data (including joint, marginal, and conditional relative frequencies). Recognize possible associations and trends in the data.		
Explanations	None		
Content Limits	This standard is aligned to Algebra I only. Bivariate data Positive rational numbers		
Common Item Formats	The Item Formats section on pages 11 through 13 provides a list of item formats that may be used to assess this standard. The common item formats include but are not limited to those shown with the sample task demands.		
Context	Context is allowed.	**Math Practices**	1, 2, 3, 4, 5, 8
Sample Task Demands		**Common Item Formats**	**Recommended Math Practices**
Students will be required to construct a contingency table in order to show the relationships between variables.		• Equation Response • Multiple Choice Response • Table Response	1, 2, 4, 5, 8
Students will be required to interpret tables to calculate marginal and joint frequencies within the context.			1, 2, 4, 5, 8
Students will be required to identify patterns in a distribution in order to answer questions pertaining to the data set and context.			1, 2, 3, 4, 5, 8

Content Standards	AzCCRS.Math.Content.S-ID.B.6 Represent data on two quantitative variables on a scatter plot, and describe how the variables are related. AzCCRS.Math.Content.S-ID.B.6a Fit a function to the data; use functions fitted to data to solve problems in the context of the data. AzCCRS.Math.Content.S-ID.B.6b Informally assess the fit of a function by plotting and analyzing residuals. AzCCRS.Math.Content.S-ID.B.6c Fit a linear function for a scatter plot that suggests a linear association.
Explanations	The residual in a regression model is the difference between the observed and the predicted y for some x (y the dependent variable and x the independent variable). So if we have a model y = ax + b and a data point (xi, yi), the residual is for this point is ri = yi - (axi+ b) . Students may use spreadsheets, graphing calculators, and statistical software to represent data, describe how the variables are related, fit functions to data, perform regressions, and calculate residuals.
Content Limits	This standard is aligned to Algebra I. Only S-ID.B.6a is aligned to Algebra II (fitting functions for Algebra I and using functions for Algebra II). Rational numbers; Bivariate data; Linear, quadratic, and exponential models for 6a
Common Item Formats	The Item Formats section on pages 11 through 13 provides a list of item formats that may be used to assess this standard. The common item formats include but are not limited to those shown with the sample task demands.

Context	Context is not allowed.	**Math Practices**	2, 3, 4, 5, 7, 8

Sample Task Demands	**Common Item Formats**	**Recommended Math Practices**
Students will be required to select a function that best represents the data given a set of data. (a)	• Equation Response • Graphic Response • Multiple Choice Response	2, 4, 5, 7, 8
Students will be required to plot and analyze residuals on a number line. (b)		2, 4, 5, 7, 8
Students will be required to create a linear function that best represents the data given a scatter plot. (c)		2, 3, 4, 5, 7, 8

<table>
<tr><td>Content Standards</td><td colspan="3">AzCCRS.Math.Content.S-ID.C.7 Interpret the slope (rate of change) and the intercept (constant term) of a linear model in the context of the data.</td></tr>
<tr><td>Explanations</td><td colspan="3">None</td></tr>
<tr><td>Content Limits</td><td colspan="3">This standard is aligned to Algebra I only.
A linear model should be provided
The model should not fit exactly a set of data, if given</td></tr>
<tr><td>Common Item Formats</td><td colspan="3">The Item Formats section on pages 11 through 13 provides a list of item formats that may be used to assess this standard. The common item formats include but are not limited to those shown with the sample task demands.</td></tr>
<tr><td>Context</td><td>Context is required.</td><td>Math Practices</td><td>1, 2, 4, 5, 6</td></tr>
<tr><td colspan="2">Sample Task Demands</td><td>Common Item Formats</td><td>Recommended Math Practices</td></tr>
<tr><td colspan="2">Students will be required to interpret the rate of change and/or constant term of a linear model to identify valid conclusions.</td><td rowspan="2">• Equation Response
• Multiple Choice Response
• Multi-Select Response</td><td>1, 2, 4, 5</td></tr>
<tr><td colspan="2">Students will be required to identify the value in a linear model that represents a given interpretation.</td><td>1, 2, 4, 5, 6</td></tr>
</table>

<table>
<tr><td>Content Standards</td><td colspan="3">AzCCRS.Math.Content.S-ID.C.8 Compute (using technology) and interpret the correlation coefficient of a linear fit.</td></tr>
<tr><td>Explanations</td><td colspan="3">None</td></tr>
<tr><td>Content Limits</td><td colspan="3">This standard is aligned to Algebra I only.
Items should focus on interpreting a given correlation coefficient</td></tr>
<tr><td>Common Item Formats</td><td colspan="3">The Item Formats section on pages 11 through 13 provides a list of item formats that may be used to assess this standard. The common item formats include but are not limited to those shown with the sample task demands.</td></tr>
<tr><td>Context</td><td>Context is required.</td><td>Math Practices</td><td>4, 5, 8</td></tr>
<tr><td colspan="2">Sample Task Demands</td><td>Common Item Formats</td><td>Recommended Math Practices</td></tr>
<tr><td colspan="2">Students will be required to interpret the correlation coefficient of a linear fit.</td><td rowspan="2">• Equation Response
• Multiple Choice Response</td><td>4, 5, 8</td></tr>
<tr><td colspan="2">Students will be required to identify another correlation coefficient that satisfies a given condition given a correlation coefficient (i.e., a coefficient that shows a better positive correlation than 0.7).</td><td>4, 5, 8</td></tr>
</table>

Content Standards	AzCCRS.Math.Content.S-ID.C.9 Distinguish between correlation and causation.		
Explanations	Some data leads observers to believe that there is a cause and effect relationship when a strong relationship is observed. Students should be careful not to assume that correlation implies causation. The determination that one thing causes another requires a controlled randomized experiment.		
Content Limits	This standard is aligned to Algebra I only. Bivariate, linear data Items should focus on the fact that causation cannot be determined from correlation, rather than asking the student to decide which relationships are causal and which are not.		
Common Item Formats	The Item Formats section on pages 11 through 13 provides a list of item formats that may be used to assess this standard. The common item formats include but are not limited to those shown with the sample task demands.		
Context	Context is required.	**Math Practices**	3, 4, 6
Sample Task Demands		**Common Item Formats**	**Recommended Math Practices**
Students will be required to distinguish information that a correlation coefficient provides (fit, trend) to information it does not (causation).		• Multiple Choice Response • Multi-Select Response	3, 4, 6